On quark masses

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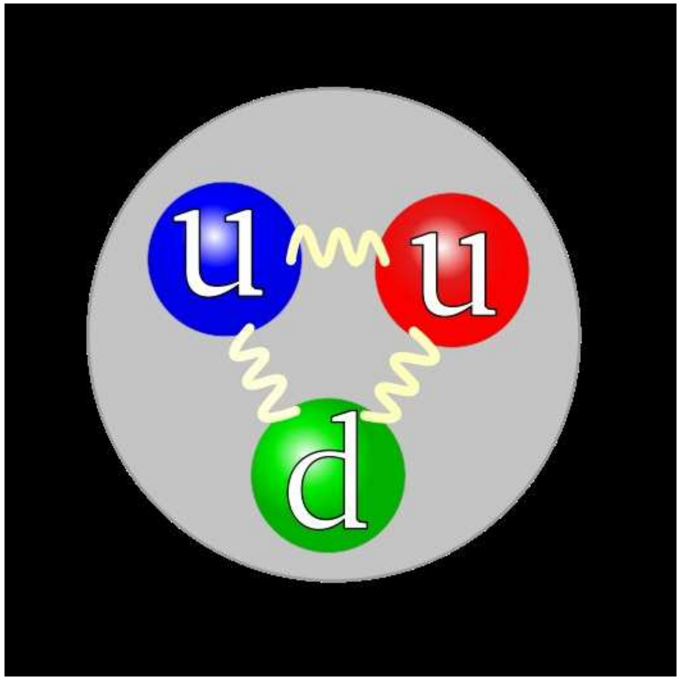
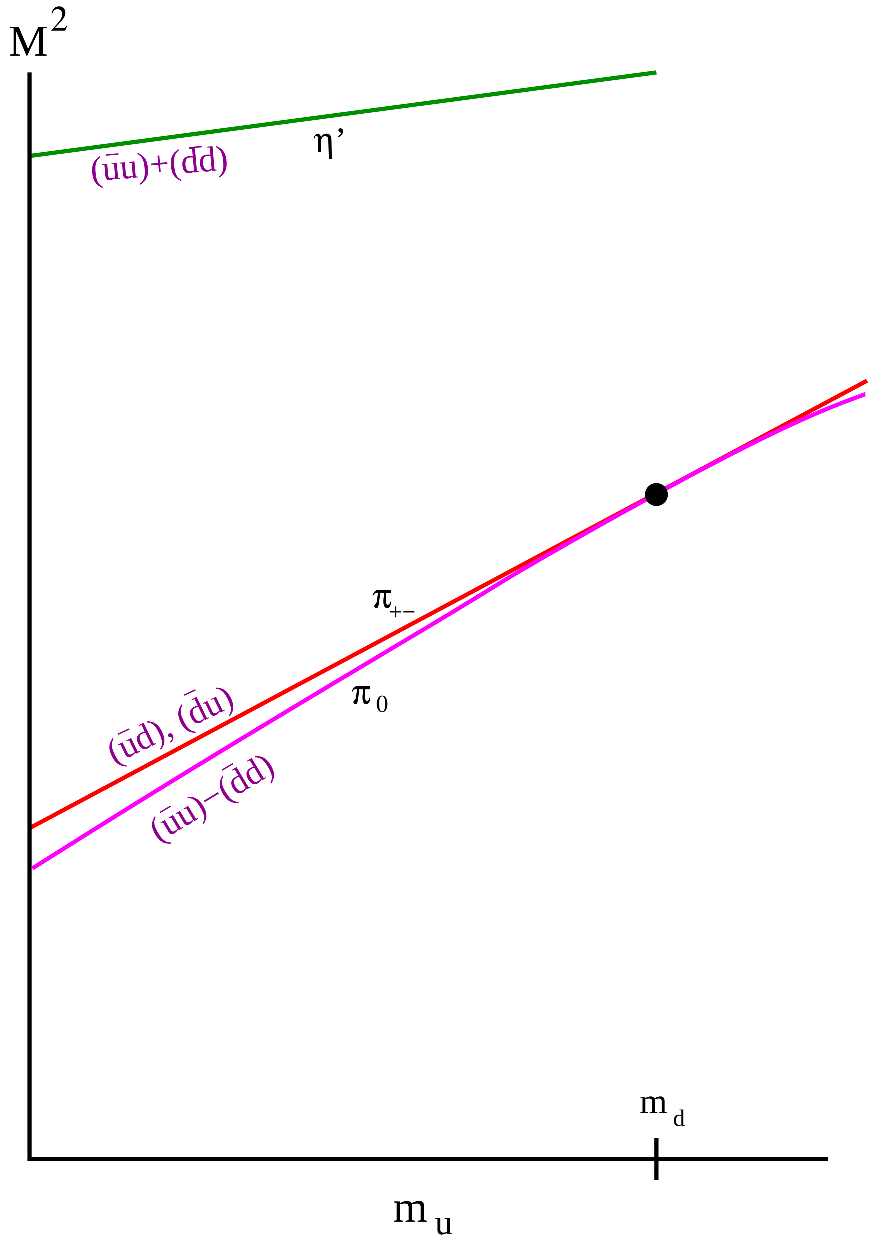
BNL



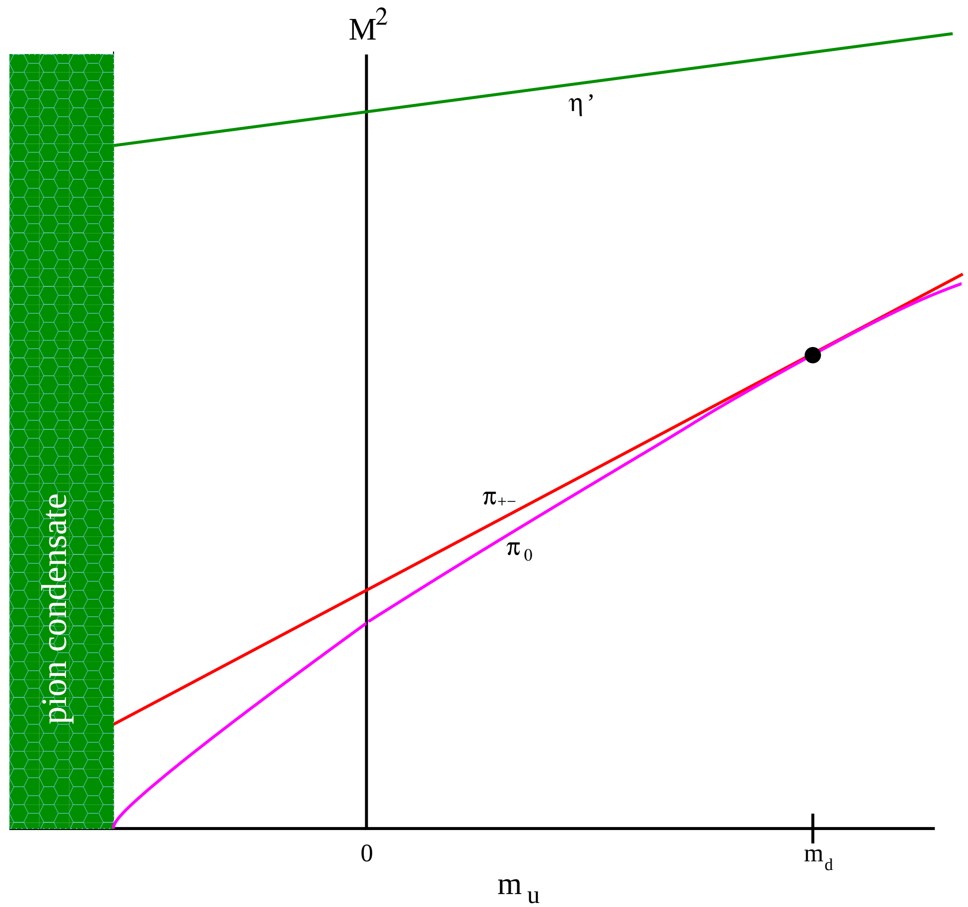
Pseudoscalars in two flavor QCD

- fix m_d , vary m_u
  - $M_\pi^2 \propto \frac{m_u+m_d}{2}$
  - $M_{\eta'} \sim \Lambda_{qcd}$
- with isospin broken
  - $M_{\pi_\pm}^2 - M_{\pi_0}^2 \propto (m_d - m_u)^2$
  - η', π_0, glueballs all mix



No singularity at $m_u = 0$

- extrapolate to negative m_u
- $M^2_{\pi_0}$ can go negative
- pion condensate forms
  - $\langle \pi_0 \rangle \neq 0$
  - CP broken
- occurs at $\Theta = \pi$
  - $\prod_q m_q < 0$



Dashen 1971

Manifested in both "linear" and "nonlinear" sigma models

Second order transition at non-vanishing m_u and m_d of opposite sign

- long distance physics without small Dirac eigenvalues

No structure at $m_u = 0$ when $m_d \neq 0$

- no long distance physics despite possible small Dirac eigenvalues

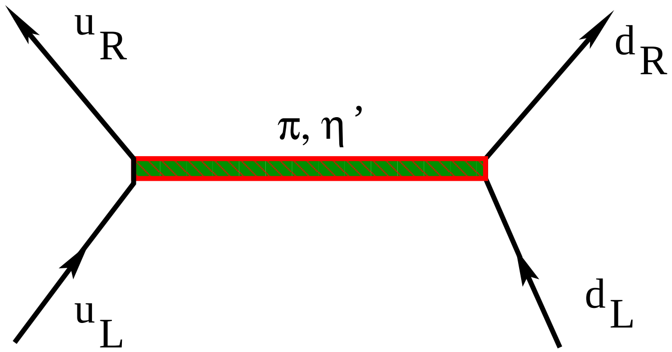
At the heart of several frustrating and bitter controversies

- Does $m_u = 0$ have any fundamental meaning?
- Do rooted staggered fermions make sense?
- Is topological susceptibility a physical observable?

Two flavors in the massless limit: $m_u = m_d = 0$

- massive proton, neutron, eta prime, glueballs
- 3 massless Goldstone pions

Eta prime and neutral pion: distinct mixtures of $\overline{u}u$, $\overline{d}d$, and glue

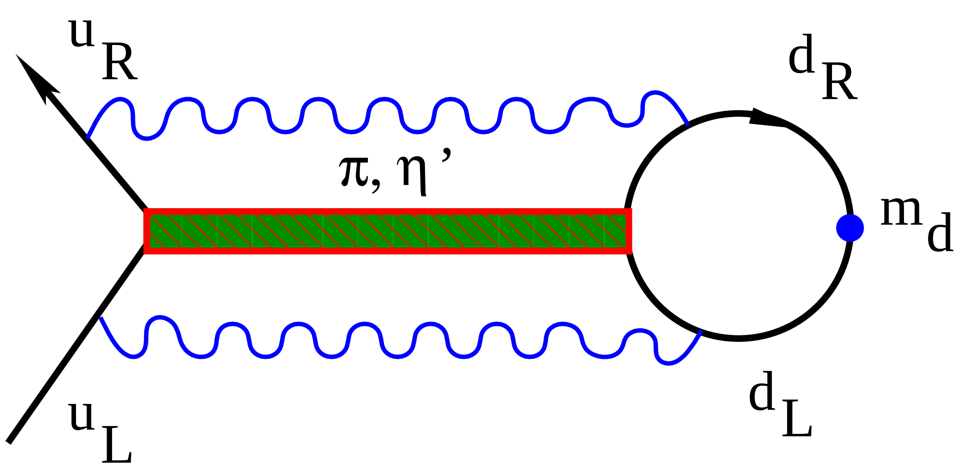


- anomaly: π_0 and η' not degenerate
- four point vertex $\langle \overline{u}_L u_R \, \overline{d}_L d_R \rangle$ does not vanish

Helicity-flip quark-quark scattering does not vanish in the chiral limit

Now turn on a small d quark mass

- closing d loop induces $u_L\ u_R$ mixing



- gluons inserted to compensate for odd meson parity

Non-zero d quark mass induces an effective mass for the u quark

Non-perturbative effects

- renormalize $\frac{m_u}{m_d}$
- quark mass ratios not renormalization group invariant
  - (except in isospin limit)

Effect automatically included in lattice simulations

Old point

- Georgi, McArthur, 1981 (unpublished)
- Banks, Nir, Seiberg, 1994 (conference proceedings)
- MC, 2003 (unpublished)
- MC, 2004 (PRL)

Intense consternation from the perturbative community

- effect not seen perturbatively, i.e. in the $\overline{MS}$ scheme
- consequences
  - mass renormalization is not flavor blind
  - mass independent regularization problematic
  - inherent ambiguities defining $m_u = 0$

$\overline{MS}$ is only a perturbative regulator

- when $m_u \neq m_d$

Matching lattice masses to $\overline{MS}$ is not appropriate!

Specific critiques

Complaint 1:

- Use a mass independent regularization
  - $a \frac{dm_i}{da} = \gamma(g) m_i \;\Rightarrow\; \frac{m_i}{m_j} = \text{constant}$

Response:

- allowed, but obscures above off-diagonal m_d effect on m_u
- no guarantee that $\frac{m_i}{m_j}$ universal between schemes
- lattice is not a mass independent scheme
  - unclear how to do matching

When $m_u \neq m_d$

- isospin broken
  - $\frac{M^2_{\pi^0}}{M^2_{\pi^\pm}} = 1 - O\left(\frac{(m_u - m_d)^2}{(m_u + m_d)\Lambda_{qcd}}\right)$

Holding quark mass ratios fixed

- hadronic mass ratios scale dependent

Holding hadronic mass ratios fixed

- quark mass ratios scale dependent

Complaint 2:

- Do matching at 100 GeV
- instantons exponentially suppressed and irrelevant

Response:

- the lattice simulations are not done at miniscule scales
  - instanton effects must be included
- $1/g^2 \sim \log(\mu) \sim \log(1/a)$
  - exponential suppression in $1/g^2 \ \rightarrow$ power in scale μ

Effect controlled by

- $M_{\eta'} - M_{\pi_0} \propto \mu\, g^{-\beta_1/\beta_0^2}\, e^{-1/(2\beta_0 g^2)} \not\to 0$
  - $\beta_0 = \frac{1}{16\pi^2}(11 - 2N_f/3)$
  - $\beta_1 = \left(\frac{1}{16\pi^2}\right)^2 (102 - 38N_f/3)$
- also proportional to $m_d - m_u$
- estimate at scale $\mu = 2$ GeV
  - $\Delta m_u(\mu) \sim \frac{(M_{\eta'} - M_{\pi_0})\ (m_d - m_u)}{\mu} = O(1 \text{ MeV})$
  - same magnitude as quoted “results”

Note

- $M_{\eta'} \propto \mu\, g^{-\beta_1/\beta_0^2}\, e^{-1/(2\beta_0 g^2)}$

- exponential behavior controlled by

- $\frac{1}{2\beta_0 g^2} = \frac{8\pi^2}{(11-2n_f/3)g^2} << \frac{8\pi^2}{g^2}$ = classical instanton action

- topological excitations above quantum, not classical, vacuum

- classical instanton action strongly overestimates suppression

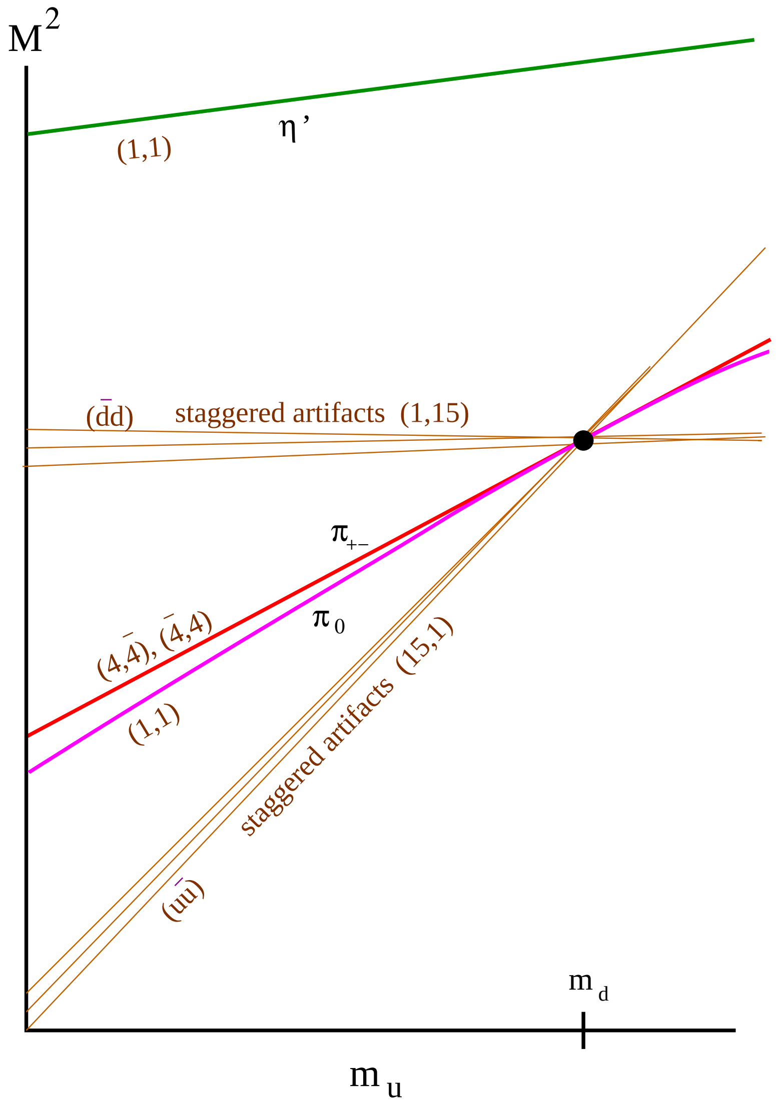
Rooted staggered quarks

- tastes: $(SU(4)_u, SU(4)_d)$
- well separated spurious states
- not only in chiral limit
- one massless at $m_u = 0$
  - required by symmetry

Can multiple artifacts cancel?

- requires unitarity violation

Plausible???



Summary

Non-perturbative effects mix mass terms for different species

- effect absent in perturbation theory
  - inappropriate to match lattice and perturbative masses

Interesting phase structure with negative mass quarks

- CP violating pion condensation
- no structure at $m_u = 0$ when $m_d \neq 0$

Crucial to resolving many controversies

  - $m_u = 0$, topological susceptibility, rooting

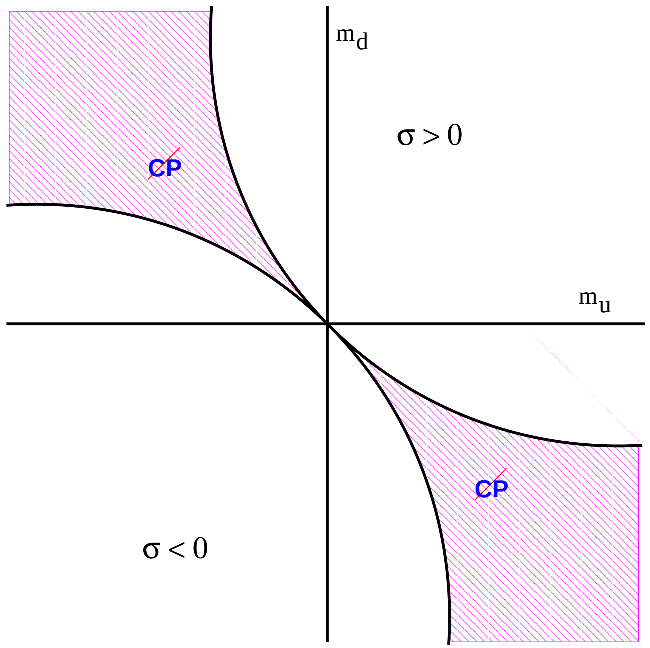
Review: Acta Physica Slovaca 61, 1 (2011), arXiv:1103.3304

free download at http://www.physics.sk/aps/

Extra Slides

Ising-like transition at $m_u < 0$

- order parameter $\langle \pi_0 \rangle \neq 0$
- breaks CP spontaneously



Connected with the anomaly and $M_{\eta'} \sim \Lambda_{qcd}$

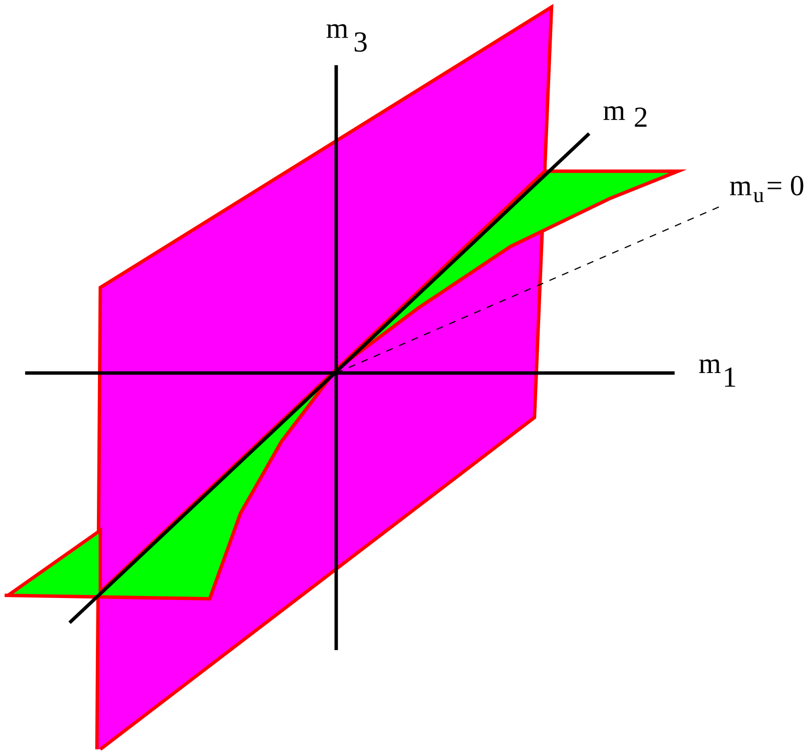
- non-perturbative

General mass term $m_1\overline{\psi}\psi + m_2\overline{\psi}\tau_3\psi + im_3\overline{\psi}\gamma_5\psi$

- average quark mass, quark mass difference, CP violation from Theta

Two intersecting first order surfaces

- $(m_1 = 0, m_3 \neq 0)$ and $(m_1 < m_2, m_3 = 0)$

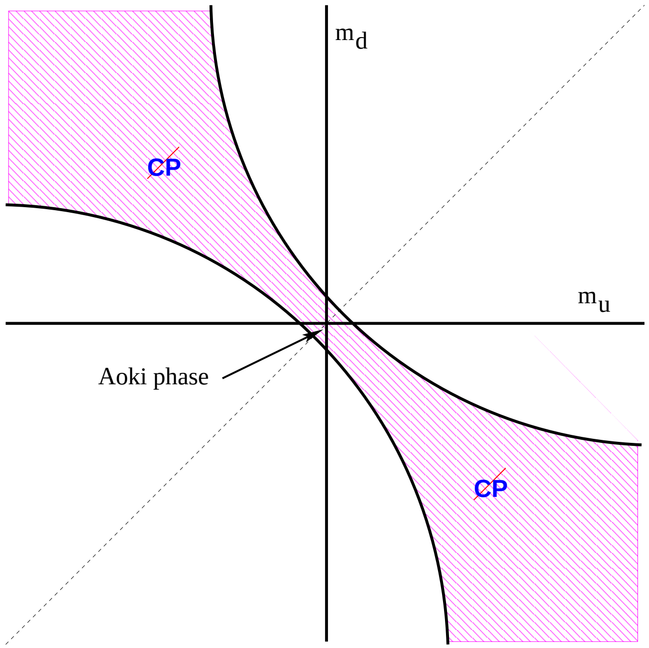


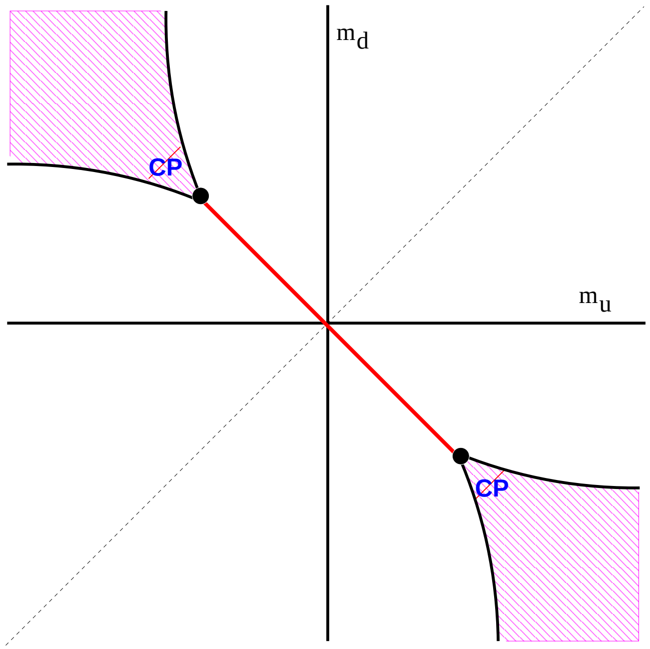
Second order edge at $m_3 = 0, 0 < |m_1| < |m_2|$

CP breaking related to the Aoki phase

- Wilson fermion lattice artifacts
- phase persists in isospin limit

- First order alternative





Which alternative remains controversial

- can depend on lattice action